

Singlet versus triplet particle-hole condensates in quantum oscillations in cuprates

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Quantum oscillations in a tilted magnetic field offer the possibility of distinguishing singlet versus triplet order parameters in the particle hole channel provided the measurements reflect a putative “normal” state of a density wave obtained by applying a high magnetic field at low temperatures. A theoretical analysis is given that compares spin density wave, a singlet d -density wave, and a triplet d -density wave. While the existence of a spin zero in the oscillation amplitude is a necessary consequence of a singlet order parameter, a triplet order parameter may or may not exhibit a spin zero, making it a quantitative issue that depends on the actual extremal orbits on the Fermi surface. Nonetheless, a theoretical analysis can shed light on the striking recent measurements in $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$.

I. INTRODUCTION

A remarkable experiment in 2007¹ involving quantum oscillations of the Hall coefficient in $\text{YBa}_2\text{Cu}_3\text{O}_{6.5}$ has raised an important question regarding the ground state, as the superconducting dome is crushed by a high magnetic field, high enough to destroy superconductivity to a large extent but not high enough on the scale of electronic energies.^{2,3} Subsequently a large number of such experiments have provided overall consistency, but perhaps not full agreement in all respects. Another striking observation has been a similar measurement in electron doped $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ where the measurements are carried out between 30–65 Tesla, far above the upper critical field H_{c2} less than 10 Tesla.⁴ If any universality is to hold, the mechanism of quantum oscillation must be the same in electron and hole doped cuprates,⁵ even though the measurements in hole doped cuprates are perhaps below H_{c2} , but unquestionably carried out in the resistive state. An emerging view is that the oscillations result from Fermi pockets formed by a suitable density wave state, a condensate in the particle-hole channel. We explore this possibility to shed light on two conflicting experiments in a tilted magnetic field, one in which no spin zeros were found^{6,7} and the other in which they were found.⁸

Condensates in particle-hole channel are fundamentally different from condensates in the particle-particle channel—superconductors. Since there is no exchange requirement between a particle and a hole, the symmetry of the orbital wave function does not constrain the symmetry of the spin wave function. A given orbital symmetry can come in both singlet and triplet varieties. These order parameters are very nicely classified on the basis of the angular momentum channel.^{9,10} Here, we concentrate on the s and d -orbital channels leaving out the possible p -channel, which probably is unlikely for the cuprates. Thus, the singlet (sDDW) and the triplet (tDDW) d -density wave order parameters are, considering only the two-dimensional and the two-fold commensurate case:

$$\langle c_\alpha^\dagger(\mathbf{k} + \mathbf{Q})c_\beta(\mathbf{k}) \rangle = i\Phi_{\mathbf{Q}}f(\mathbf{k})\delta_{\alpha\beta}, \quad (1)$$

$$\langle c_\alpha^\dagger(\mathbf{k} + \mathbf{Q})c_\beta(\mathbf{k}) \rangle = i\Phi_{\mathbf{Q}}f(\mathbf{k})\hat{n} \cdot \boldsymbol{\sigma}_{\alpha\beta}. \quad (2)$$

Here $c_\alpha(\mathbf{k})$ is the fermion destruction operator of spin index

α and the d -wave form factor is

$$f(\mathbf{k}) = (\cos k_x a - \cos k_y a)/2. \quad (3)$$

The vector $\mathbf{Q} = (\pi/a, \pi/a)$ and the magnitude $\Phi_{\mathbf{Q}}$ is real. The singlet order parameter transforms as identity in the spin space, while the triplet transforms as $\hat{n} \cdot \boldsymbol{\sigma}_{\alpha\beta}$, \hat{n} being the direction of the spin quantization axis and $\boldsymbol{\sigma}$ the standard Pauli matrix. We shall assume that spin-orbit coupling is negligibly small.

The singlet and the triplet s -wave density waves are the conventional charge (CDW) and spin density waves (SDW), defined by setting $f(\mathbf{k}) = 1$ and removing the factor of i :

$$\langle c_\alpha^\dagger(\mathbf{k} + \mathbf{Q})c_\beta(\mathbf{k}) \rangle = \Phi_{\mathbf{Q}}\delta_{\alpha\beta}, \quad (4)$$

$$\langle c_\alpha^\dagger(\mathbf{k} + \mathbf{Q})c_\beta(\mathbf{k}) \rangle = \Phi_{\mathbf{Q}}\hat{n} \cdot \boldsymbol{\sigma}_{\alpha\beta}. \quad (5)$$

Note that we have denoted the magnitudes of all the order parameters by $\Phi_{\mathbf{Q}}$ for notational simplicity.

The sDDW modulates neither charge nor spin but represents staggered circulating charge currents, as shown in Fig. 1. In this case, the order parameter breaks lattice translational

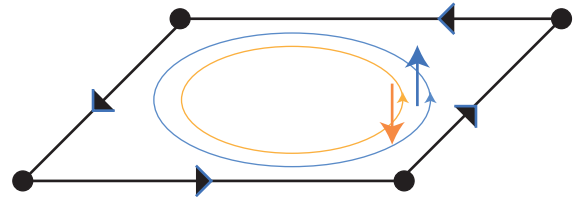


FIG. 1: (Color online) Circulating charge current for a single square plaquette corresponding to singlet DDW order parameter. Note that there is no circulating spin current.

symmetry, time reversal, parity, and a rotation by $\pi/2$, but the product of any two symmetry operations is preserved. The order parameter is “hidden” because most external probes do not couple to currents or variations of kinetic energy. The order parameter corresponding to $S = 0$, as in a sDDW, does not respond to magnetic field directly, as in a paramagnet, but the quasiparticles energies are split by the Zeeman effect.

The tDDW is further hidden because it is invariant under time reversal unlike SDW. Nonetheless both break spin rotational symmetry and lead to Goldstone modes. The tDDW, as well as the SDW, undergo a spin-flop transition in an arbitrary small magnetic field for zero spin-orbit interaction. If there is anisotropy in the spin space from spin-orbit coupling, there will be a non-zero threshold field beyond which the spin-flop transition will take place. In cuprates spin-orbit coupling is small enough that for high magnetic fields relevant for quantum oscillations, it is almost certain that the spins will be perpendicular to the applied magnetic field H , as shown in Fig. 2. There is a particularly nice way to bring out the similarities of tDDW and SDW, if we define the macroscopic order parameters by

$$\mathbf{y} = i \sum_{\mathbf{k}} f(\mathbf{k}) \sum_{\alpha, \beta} \sigma_{\beta\alpha} \langle c_{\alpha}^{\dagger}(\mathbf{k} + \mathbf{Q}) c_{\beta}(\mathbf{k}) \rangle, \quad (6)$$

$$\mathbf{m} = \sum_{\mathbf{k}} \sum_{\alpha, \beta} \sigma_{\beta\alpha} \langle c_{\alpha}^{\dagger}(\mathbf{k} + \mathbf{Q}) c_{\beta}(\mathbf{k}) \rangle. \quad (7)$$

Note the trace operation in the above definitions. The tDDW and the spin flop in the presence of a magnetic field is shown in Fig. 2

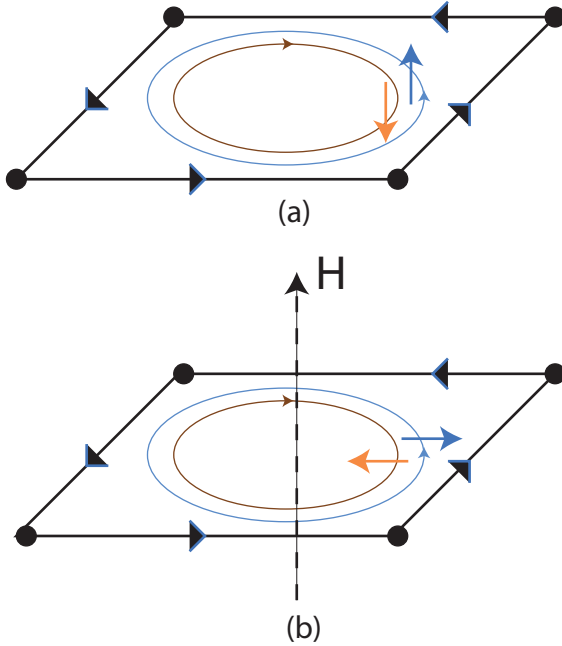


FIG. 2: (Color online) Spin-flop of triplet DDW in a magnetic field H pictured for a single square CuO plaquette with O atoms omitted. There is circulating spin current but no charge current. Here (a) corresponds to the situation when $H = 0$ and (b) to $H \neq 0$.

II. SPIN ZEROS

In this section we shall calculate the spin zeros in quantum oscillations following the Lifshitz-Kosevich (LK) formalism,

which we do not duplicate here. Only the relevant aspects pertaining to the reconstructed Fermi surfaces due to sDDW, tDDW and SDW will be described. In addition, we restrict ourselves to two dimensions without c -axis warping and bilayer splitting in $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$.¹² This simplicity reveals the essential elements without unnecessary clutter. The extension to include the neglected effects are entirely straightforward. However, we do need to specify the various parameters needed for the illustrative calculations, and these are summarized in the Appendix A. In this appendix we also provide the frequencies in the absence of the Zeeman field.

A. Singlet

The spectra of excitations for sDDW can be obtained from the following Hamiltonian

$$\mathbb{H} = \sum_{\mathbf{k} \in \text{RBZ}} \Psi_{\mathbf{k}}^{\dagger} \mathbb{A}_{\mathbf{k}} \Psi_{\mathbf{k}}, \quad (8)$$

where the reduced Brillouin zone (RBZ) is bounded by $k_y \pm k_x = \pm\pi/a$ and

$$\mathbb{A} = \begin{pmatrix} \epsilon_{\uparrow, \mathbf{k}} & iW_{\mathbf{k}} & 0 & 0 \\ -iW_{\mathbf{k}} & \epsilon_{\uparrow, \mathbf{k}+\mathbf{Q}} & 0 & 0 \\ 0 & 0 & \epsilon_{\downarrow, \mathbf{k}} & iW_{\mathbf{k}} \\ 0 & 0 & -iW_{\mathbf{k}} & \epsilon_{\downarrow, \mathbf{k}+\mathbf{Q}} \end{pmatrix}. \quad (9)$$

Here $\epsilon_{\uparrow, \mathbf{k}} = \epsilon_{\mathbf{k}} + \frac{g}{2}\mu_B H$ and $\epsilon_{\downarrow, \mathbf{k}} = \epsilon_{\mathbf{k}} - \frac{g}{2}\mu_B H$, where μ_B is the Bohr magneton. The four component spinor is given by $\Psi_{\mathbf{k}}^{\dagger} = (c_{\mathbf{k}, \uparrow}^{\dagger}, c_{\mathbf{k}+\mathbf{Q}, \uparrow}^{\dagger}, c_{\mathbf{k}, \downarrow}^{\dagger}, c_{\mathbf{k}+\mathbf{Q}, \downarrow}^{\dagger})$. Of course the up and the down spin sectors are decoupled, and the eigenvalues are

$$\lambda_{\sigma, \mathbf{k}}^s = \frac{[\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k}+\mathbf{Q}}]}{2} \pm \sqrt{\frac{[\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{Q}}]^2}{4} + W_{\mathbf{k}}^2} + \frac{g}{2}\sigma\mu_B H, \quad (10)$$

which clearly shows Zeeman splitting as for a free spin. In what follows, to simplify the notation, we shall define:

$$\epsilon_1 \equiv \frac{1}{2}[\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{Q}}] \quad (11)$$

$$\epsilon_2 \equiv \frac{1}{2}[\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k}+\mathbf{Q}}] \quad (12)$$

and drop the wave vector arguments when there is not any possibility of confusion. For notational simplicity, we denote $\sigma = +1 \equiv \uparrow$ and $\sigma = -1 \equiv \downarrow$.

The Zeeman term depends on the total magnetic field. It is included exactly within the Hamiltonian, because it is crucial in determining the sensitive interference of the amplitudes of quantum oscillations related to spin. The effect of the magnetic field on the orbital part is treated differently. First, its effect on the DDW order parameter can be neglected, as long as the system is deep inside the DDW phase; the fields relevant to quantum oscillations are energetically weak perturbations on the DDW gap.¹³ Clearly, close to a quantum phase transition where the DDW gap collapses, this will no longer be true—a situation that is not relevant to the experiments addressed here. Second, in calculating quantum oscillations using LK formalism only the extremal Fermi surface areas and

its various derivatives are needed, which can be calculated in the absence of the orbital part of the magnetic field. The oscillations arising from the extremal in-plane orbits are of course determined by only the normal component H_z ; for simplicity we are ignoring warping and bilayer splitting. At the wave vector $(\pi/2a, \pi/2a)$, and at symmetry related points in the Brillouin zone, there are nodes. If the chemical potential, μ , is placed at these nodes, the orbital diamagnetic susceptibility diverges as $H \rightarrow 0$ in the absence of any interlayer coupling and at zero temperature, $T = 0$.¹⁰ However, the values of μ considered here are too far from the nodal points for this to be relevant.

From the LK formalism it is immediately obvious that the spin interference factor, R_s , within the LK formalism is

$$R_s = \cos \left(\pi p \frac{m^*}{m} \frac{g}{2} \frac{1}{\cos \theta} \right), \quad (13)$$

where $\cos \theta$ is the angle between the magnetic field with respect to the normal and p stands for the p -th harmonic. Within our mean field theory $g = 2$. The singlet DDW order parameter must exhibit spin zeros corresponding to elementary fermionic excitations of charge e , spin $1/2$, $g = 2$, and the cyclotron mass m^* calculated from Eq. 10. Nonetheless, the system is not a paramagnet because the spectra in Eq. 13 correspond to a broken symmetry state far from a conventional paramagnet but with total spin $S = 0$. Within Fermi liquid theory, with the reconstructed Fermi surface, Eq. 13 should hold to all orders in perturbation theory. However, residual Fermi liquid interactions between the quasiparticles can certainly lead to renormalization of m^* and g . However, quite generally, within the Fermi liquid formalism, the residual interactions can only increase m^* and g , assuming small spin orbit coupling, which appears to be the case; see Ref. 8 and reference therein. Thus the case for the existence of spin zeros will be stronger if the Fermi liquid corrections are taken into account.

B. Triplet

In the triplet case the spin orientation is chosen to be perpendicular to the direction of the applied field because of the spin flop. Thus, with no loss of generality, we can choose $H_z = H \cos \theta$, $H_x = H_y = 0$ and $n_x = 1$, $n_y = n_z = 0$. Then the matrix \mathbb{A} is

$$\mathbb{A} = \begin{pmatrix} \epsilon_{\uparrow, \mathbf{k}} & 0 & 0 & iW \\ 0 & \epsilon_{\uparrow, \mathbf{k}+\mathbf{Q}} & -iW & 0 \\ 0 & iW & \epsilon_{\downarrow, \mathbf{k}} & 0 \\ -iW & 0 & 0 & \epsilon_{\downarrow, \mathbf{k}+\mathbf{Q}} \end{pmatrix} \quad (14)$$

The eigenvalues are

$$\lambda_{\sigma, \mathbf{k}}^t = \epsilon_2 \pm \sqrt{(\epsilon_1 + \frac{g}{2} \sigma \mu_B H)^2 + W^2} \quad (15)$$

The four eigenvectors are various linear superpositions involving the coherence factors of the original fermion operators and

do not have definite spin unlike the singlet case. For arbitrary \mathbf{k} the mixing depends on the \mathbf{k} -space orbit, and this gives rise to a dynamically generated spin-orbit effect, as can be seen from the quasiparticle spectra, if we expand in powers of the magnetic field. The differences between the energies of the two close energy levels, either the holes or the electrons, are given by

$$\Delta_h = \Delta_e \approx g \mu_B H \frac{|\epsilon_1|}{\sqrt{\epsilon_1 + W^2 + (g \mu_B H/2)^2}}, \quad (16)$$

$$\equiv g_{\text{eff}}(\mathbf{k}) \mu_B H, \quad (17)$$

that is, by an effective g -factor. A typical plot of g_{eff} is shown in Fig. 3. Another way to picture the distinction between the

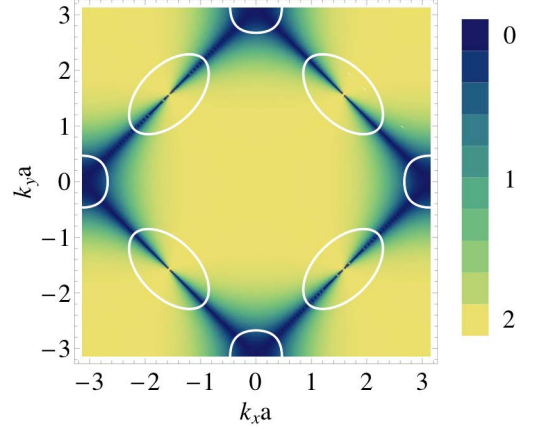


FIG. 3: (Color online) A map of $g_{\text{eff}}(\mathbf{k})$ for tDDW. Note that it is mostly 2 for most of the hole pocket and mostly zero for the electron pocket. This effective g -factor will split the hole and the electron pockets into two close pockets depending on the magnetic field. The actual splitting due to Zeeman coupling is not visible on the scale of the plot.

singlet and the triplet cases is the schematic energy level diagram shown in Fig. 4.

1. Effective g -factor

Let us focus on the electron pockets; the results for the hole pockets follow identically. Let

$$\epsilon_{\pm} = \epsilon_2 + \left[\left(\epsilon_1 \pm \frac{g}{2} \mu_B H \right)^2 + W^2 \right]^{1/2} \quad (18)$$

To calculate the interference factor due to the two close electron pockets within the LK formalism, we must compute the sum

$$\begin{aligned} \sum_{j=\pm} e^{2\pi i p n_{mj}} &= e^{2\pi i p n_{m+}(\mu)} e^{2\pi i p \left(\frac{\partial n_{m+}}{\partial \epsilon_+} \right)_{\mu} (\epsilon_+ - \mu)} \\ &+ e^{2\pi i p n_{m-}(\mu)} e^{2\pi i p \left(\frac{\partial n_{m-}}{\partial \epsilon_-} \right)_{\mu} (\epsilon_- - \mu)}. \end{aligned} \quad (19)$$

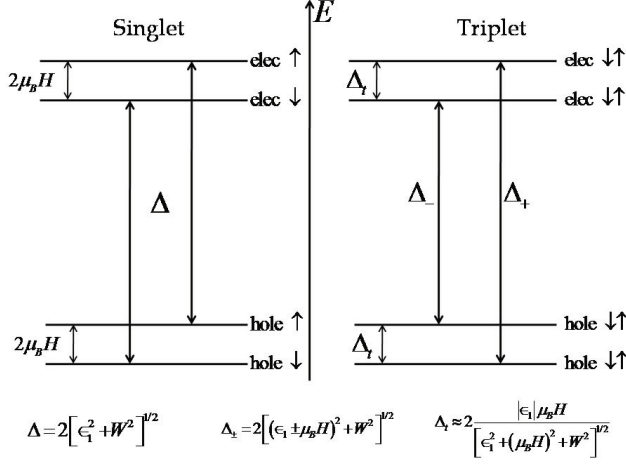


FIG. 4: The splitting of the levels in a magnetic field. Note that in the triplet case we cannot identify the levels with a definite spin quantum number. The hole and the electron pockets are labelled “hole” and “elec”.

corresponding to the Landau level index $n_{m\pm}$ corresponding to the extremal Fermi surface orbits. Since the splitting is small for experimentally relevant H , it is a good approximation to assume that $n_{m+}(\mu) \approx n_{m-}(\mu) \approx \bar{n}_m(\mu)$. Similarly,

$$\left(\frac{\partial n_{m+}}{\partial \varepsilon_+}\right)_\mu \approx \left(\frac{\partial n_{m-}}{\partial \varepsilon_-}\right)_\mu \approx \left(\frac{\partial \bar{n}_m}{\partial \bar{\varepsilon}}\right)_\mu, \quad (20)$$

where $\bar{n}_m = (n_{m+} + n_{m-})/2$, $\varepsilon_+ = \bar{\varepsilon} + \Delta\varepsilon$, and $\varepsilon_- = \bar{\varepsilon} - \Delta\varepsilon$. Thus,

$$\sum_j e^{2\pi i p n_{mj}} \approx e^{2\pi i p \bar{n}_m(\mu)} \exp\left(2\pi i p \left(\frac{\partial \bar{n}_m}{\partial \bar{\varepsilon}}\right)_\mu (\bar{\varepsilon} - \mu)\right) \times \cos\left(2\pi p \left(\frac{\partial \bar{n}_m}{\partial \bar{\varepsilon}}\right)_\mu \Delta\varepsilon\right). \quad (21)$$

We can set $(\bar{\varepsilon} - \mu) \approx 0$ and the identification of $\bar{\varepsilon}$ and $\Delta\varepsilon$ follows trivially from Eq. 18. Then the interference factor is

$$R_s = \cos\left(\pi p \frac{m^*}{m_e} \frac{g_{\text{eff}}(\mathbf{k})}{2} \frac{1}{\cos\theta}\right) \quad (22)$$

As the g_{eff} is \mathbf{k} -dependent we average over the extremal orbits shown in Fig. 3.

2. Filling of the Landau levels

There is another method to obtain the interference factor that serves as a consistency check, especially because we have made approximations. In this approach we focus on

the filling of the Landau levels. We can define the average $\bar{n} = \frac{1}{2}(n_+ + n_-)$ and the difference $\Delta n = \frac{1}{2}(n_+ - n_-)$. Where the plus sign makes reference to the pocket that is bigger and the minus sign to the pocket that is smaller. It does not matter if the difference is due to the Zeeman spin splitting, as for the singlet case, or the splitting in the spectra in the triplet case. If we extend this notation to the extremal areas m , $\bar{n}_m = \frac{1}{2}(n_{m+} + n_{m-})$ and the difference $\Delta n_m = \frac{1}{2}(n_{m+} - n_{m-})$. Again taking advantage of small splitting, we can show that in the LK formula

$$\sum_{j=\pm} e^{2\pi i p n_{mj}} = e^{2\pi i p \bar{n}_m(\mu)} \cos(2\pi p \Delta n_m) \quad (23)$$

Now the dependences on the g_{eff} , cyclotron masses, and the tilt angle are contained in Δn_m . If we compare with the previous results we obtained for the cosine factor we get

$$\cos\left(\pi p \frac{g_{s,t}}{2} \frac{m^*}{m_e} \frac{1}{\cos\theta}\right) = \cos(2\pi p \Delta n_m). \quad (24)$$

valid for both singlet and triplet. If $\Delta n_m = 0$ the argument vanishes and the cosine is unity; there will be no spin zeros at all. We can deduce what is the effective g -factor for the triplet if we know the result for the singlet: $(\Delta n_m)_t / (\Delta n_m)_s = g_t m_t^* / g_s m_s^*$. Assuming that the effective cyclotron masses are approximately the same (see Table I) for sDDW and tDDW, we get

$$g_t \approx g_s \frac{(\Delta n_m)_t}{(\Delta n_m)_s}. \quad (25)$$

TABLE I: Calculated cyclotron masses in units of the free electron mass for sDDW, tDDW, and SDW order parameters. e-p and h-p correspond respectively to electron and hole pockets, and the two rows to the two distinct Fermi surfaces. The errors are less than one part in 1000. The first row corresponds to the inner pocket and the second to the outer pocket.

sDDW	sDDW	tDDW	tDDW	SDW	SDW
e-p	h-p	e-p	h-p	e-p	h-p
1.55	0.95	1.56	0.96	1.27	1.12
1.55	0.97	1.54	0.96	1.28	1.13

It is easy to see that that n_m is proportional to the extremal Fermi surface areas. For our purposes we can set $g_s = 2$ to compute g_t from the integration of the Fermi surface areas. The results are shown below in Table II. The absolute value of R_s , the factor that is responsible for the spin zeros is shown for the electron pocket, as a function of the tilt of the magnetic field with respect to the normal in Fig. 5. In the singlet case we have chosen the free electron value of $g_s = 2$. In the triplet case the calculated effective $g_t = 0.32$ is used for the electron pocket. We can see that for the singlet a spin zero is found close to 50° , while the first spin zero for the triplet occurs above 60° . Similar results are shown for the hole pocket in Fig. 6 using $g_t = 1.65$.

TABLE II: The effective g -factors. The computations were carried out for 40 Tesla field for illustrative purposes. The Method 1 is based on $g_{\text{eff}}(\mathbf{k})$ in Eq. 22 and the Method 2 is based on Eq. 25. A reasonable estimate of the errors may be taken to be the difference of the results between the two methods.

g_{eff}	tDDW	tDDW	SDW	SDW
	e-p	h-p	e-p	h-p
Method 1	0.32	1.65	0.45	1.42
Method 2	0.31	1.65	0.40	1.42

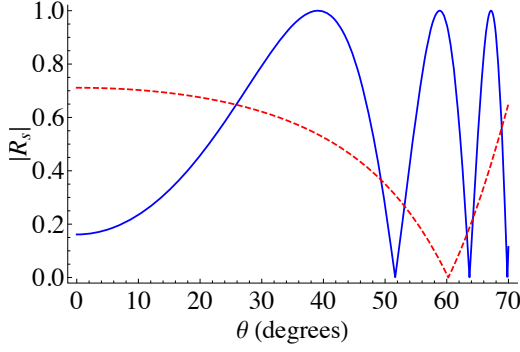


FIG. 5: (Color online) $|R_s|$ as a function of the angle of the tilted magnetic field for the electron pocket and for $p = 1$. The solid line corresponds to sDDW and the dashed the tDDW.

C. Commensurate SDW

A commensurate SDW in our scheme is an orbital s -wave order parameter for which the calculation proceeds identically except that the $f(\mathbf{k}) = 1$. The results are shown in Fig. 7.

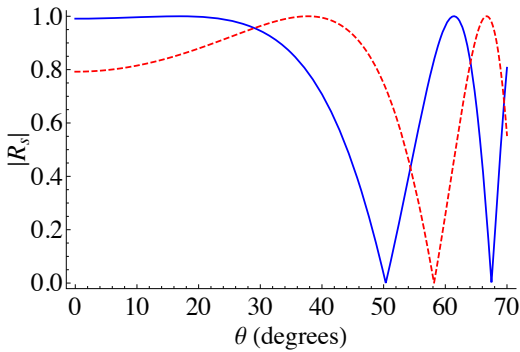


FIG. 6: (Color online) $|R_s|$ as a function of the angle of the tilted magnetic field for the hole pocket and $p = 1$. The solid line corresponds to sDDW and the dashed the tDDW.

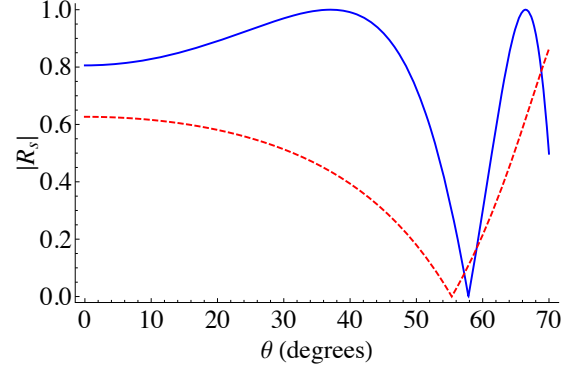


FIG. 7: (Color online) $|R_s|$ as a function of the angle of the tilted magnetic field for SDW and $p = 1$. The solid line in this figure represents the hole pocket and the dashed line the electron pocket.

III. CONCLUSION

The rigorous statement we can make is that a singlet order parameter will definitely lead to Zeeman splitting (see Eq. 10) and therefore spin zeros (recall the discussion of the g -factor above). A triplet order parameter, be it SDW or triplet-DDW, does not exhibit Zeeman splitting (see Eq. 15). It does lead to some dependence of the amplitude of oscillations on the tilt in a non-universal manner, and may or may not lead to spin zeros within a reasonable angular range, such that the system is not driven into the superconducting state, which may lead to the loss of the amplitude of the oscillations. In other words, the experiments in Ref. 8 are consistent with a singlet order parameter.

It is remarkable that once the conventional band parameters are adopted, and the oscillation frequency is adjusted close to the experimentally observed one at 526 T for the electron pocket (for the other frequency, effects such as bilayer splitting and c -axis warping, must be taken into account), the oscillation amplitude as a function of the tilt angle for the singlet order parameter is very close to the measured amplitude in Ref. 8. As one can see from the plots, the triplet order parameter behaves very differently.

The spin zeros determine only the product gm^*/m_e , which calculated in the case of sDDW for the electron pocket is 3.1, as opposed to 3.2 in Ref. 8. For the hole pocket, in contrast, it is calculated to be 1.92. Unfortunately there does not appear to be any evidence of the hole pockets in experiments, about which we have commented elsewhere.¹⁴

A useful insight into tDDW can be obtained from the coherence factors corresponding to the four eigenvectors of the matrix in Eq. 14. These are

$$\begin{aligned}
 \gamma_{1,\mathbf{k}} &= v_{1,\mathbf{k}} c_{\mathbf{k},\uparrow} + u_{1,\mathbf{k}} c_{\mathbf{k}+\mathbf{Q},\downarrow}, \\
 \gamma_{2,\mathbf{k}} &= v_{2,\mathbf{k}} c_{\mathbf{k}+\mathbf{Q},\uparrow} + u_{2,\mathbf{k}} c_{\mathbf{k},\downarrow}, \\
 \gamma_{3,\mathbf{k}} &= v_{3,\mathbf{k}} c_{\mathbf{k}+\mathbf{Q},\uparrow} + u_{3,\mathbf{k}} c_{\mathbf{k},\downarrow}, \\
 \gamma_{4,\mathbf{k}} &= v_{4,\mathbf{k}} c_{\mathbf{k},\uparrow} + u_{4,\mathbf{k}} c_{\mathbf{k}+\mathbf{Q},\downarrow},
 \end{aligned}$$

where the coherence factors are

$$\left\{ \begin{array}{l} |u_{1,\mathbf{k}}|^2 = |v_{4,\mathbf{k}}|^2 \\ |v_{1,\mathbf{k}}|^2 = |u_{4,\mathbf{k}}|^2 \end{array} \right\} = \frac{1}{2} \left(1 \pm \frac{\Delta\epsilon_{\mathbf{k},+}}{E_{\mathbf{k},+}} \right), \quad (26)$$

$$\left\{ \begin{array}{l} |u_{2,\mathbf{k}}|^2 = |v_{3,\mathbf{k}}|^2 \\ |v_{2,\mathbf{k}}|^2 = |u_{3,\mathbf{k}}|^2 \end{array} \right\} = \frac{1}{2} \left(1 \pm \frac{\Delta\epsilon_{\mathbf{k},-}}{E_{\mathbf{k},-}} \right) \quad (27)$$

and

$$E_{\mathbf{k},\sigma} = \left[\left(\epsilon_1 + \sigma \frac{g}{2} \mu_B H \right)^2 + W^2 \right]^{1/2}, \quad (28)$$

$$\Delta\epsilon_{\mathbf{k},\sigma} = \epsilon_1 + \frac{g}{2} \sigma \mu_B H. \quad (29)$$

The coherence factors reflect the fact that the quasiparticles do not have a definite spin. For arbitrary \mathbf{k} the mixing is not of equal amplitude.

Another amusing observation is that as far as the product gm^*/m_e is concerned both tDDW and SDW yield essentially identical answers even though the individual values for the effective g -factors and the m^* are different. For the electron pocket this product is 0.5 for tDDW and 0.51 for SDW. Similarly, for the hole pocket it is 1.59 for tDDW and 1.60 for SDW.

One might wonder if there are other possibilities of a singlet order parameter that could be a candidate broken symmetry state. In principle one cannot rule out CDW order, although such an order parameter would not have gone unnoticed in many other direct experiments. Thus, the existence of spin zeros may be consistent with sDDW, whose direct observation by its very nature may be considerably hidden.

Acknowledgments

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Note added: After our work was completed we learned of two interesting papers that appear to be complementary. In one, the problem has been addressed from the perspective of stripe physics (arXiv: 1007.1047v1) and in the other antiferromagnets are studied (arXiv:1006.0167v1).

Appendix A: Parameters

We use a common band structure,^{15,16}

$$\epsilon_{\mathbf{k}} = -2t(\cos k_x a + \cos k_y a) + 4t' \cos k_x a \cos k_y a - 2t''(\cos 2k_x a + \cos 2k_y a), \quad (A1)$$

where $t = 0.154$ eV, $t' = 0.32t$, $t'' = 0.5t'$. For sDDW and tDDW we choose^{9,10,17}

$$W_{\mathbf{k}} = \frac{W_0}{2}(\cos k_x a - \cos k_y a), \quad (A2)$$

with $W_0 = 0.9t$. The chemical potential $\mu = -0.775t$ then results in a hole doping of approximately 10.7%. For SDW the choices were $W_{\mathbf{k}} = W_0 = 0.675t$ and $\mu = -0.93t$, yielding a hole doping of approximately 10.6%. The frequencies corresponding to the Fermi surface areas in the absence of Zeeman splitting are (a) sDDW: 513 Tesla for the electron pocket and 1005 Tesla for the hole pocket; (b) tDDW: 513 Tesla for the electron pocket and 1004 Tesla for the hole pocket; (c) SDW: 539 Tesla for electron pocket and 1012 for the hole pocket.

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